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STRESS ANALYSIS OF ADHESIVELY BONDED REPAIRS TO FIBRE COMPOSITE--ETC(U)

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STRUCTURES REPORT 386

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SUMMARY

This paper describes a finite element method for analysing the behaviour of flaws in thin fibre composite sheets which are repaired with a bonded overlay. The method is an extension of the authors' previous work on the repair of metal structures. As illustrative examples the repair of a hole and a crack in various graphite/epoxy laminates is discussed and it is shown that in each case a bonded overlay of either boron/epoxy or titanium provides an effective repair.

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ABSTRACT

This paper describes a finite element method for analysing the behaviour of flaws in thin fibre composite sheets which are required with a bonded overlay. The method is an extension of the authors' previous work on the repair of metal structures. As illustrative examples the repair of a hole and a crack in various graphite/epoxy laminates is discussed and it is shown that in each case a bonded overlay of either boron/epoxy or titanium provides effective repair.

NOTATION

x,y,z	Cartesian coordinate system
τ_{xz}, τ_{yz}	Transverse shear stresses
γ_{xz}, γ_{yz}	Transverse shear strains
τ_{sx}, τ_{sy}	Adhesive shear stresses
t_s, t_a, t_0	Thickness of the sheet, adhesive and patch respectively
G'_{13}, G'_{23}	Transverse Shear moduli of the patch
$\hat{G}_{13}, \hat{G}_{23}$	Transverse shear moduli of the sheet
G_a	Shear modulus of adhesive
$u_0, v_0; u_s, v_s$	Inplane displacements of the patch and sheet respectively
w	Transverse displacement of the structure
x',y'	Axes of orthotropy of the patch
θ	Angle measured from local axis system to the material symmetry axis of the patch
x'',y''	Axes of orthotropy of the sheet
$\hat{\theta}$	Angle measured from local axis system to the material symmetry axis of the sheet
K^e	Element stiffness matrix for the adhesive
K_{1u}, K_{1p}	Mode I stress intensity factor before and after patching, respectively
K_{TU}, K_{TP}	Stress concentration factor before and after patching, respectively
δ	Vector containing the degrees of freedom of a bonded element

A

CONTENTS

1. INTRODUCTION

Advanced composite materials are finding increasing application in aerospace structures. In practice these materials may contain defects such as voids, splits, delaminations and cracks produced by either moulding, machining, fatigue or impact damage [1, 2]. These defects may act as stress raisers which could precipitate failure, depending on the loading, geometry and the "toughness" of the material. As a result attention has recently been focused on developing methods to predict the residual strength of notched composites [3, 4] and on developing repair procedures [5, 6]. In the latter area the Aeronautical Research Laboratories, Australia, have pioneered the use of adhesively bonded fibre reinforced plastic patches to repair flaws in metallic components. This procedure has been successfully used in the repair of stress corrosion cracks in the wings of Hercules aircraft, fatigue cracks in the landing wheels of Macchi aircraft and in the lower wing skins of Mirage III aircraft [7, 8, 9]. The present paper is concerned with the extension of the analytical methods developed by the authors for the analysis of the repairs to thin metal structures (e.g. wing skins) to cover the case when the structure is made of a fibre composite material. In this approach particular attention is paid to the adhesive which bonds the patch to the skin and allowance is made for the shear deformation in the skin, adhesive and patch. It is assumed that the skin is made as an orthotropic laminate and that, in the case of a composite patch, the patch is also an orthotropic laminate.

As illustrative examples the repairs of a crack in a graphite/epoxy panel, and of a hole whose diameter is the same lenght as the total crack length in the previous case are studied. As the repair material titanium sheet, boron/epoxy and graphite/epoxy laminates are considered. Various lay ups in the damaged graphite/epoxy panel are investigated. As a result of this analysis the use of an undirectional laminate is recommended for repairing cracks whilst the use of titanium or a quasi-isotropic lay up of boron/epoxy is recommended for repairing holes in composites.

2. METHOD OF ANALYSIS

Let us begin by considering a thin composite patch which is bonded to a thin sheet of fibre composite material. The x and y axes are taken in a plane parallel to the midsurface of the sheet with the z axis in the thickness direction. Under in-plane or transverse loading, shear stresses will be developed in the adhesive bond and it is reasonable to assume that these will be continuous across the adhesive-sheet interface as well as across the adhesive patch interface. Furthermore these shear stresses, τ_{xz} and τ_{yz}, are zero at a free surface or at a plane of symmetry, and it is reasonable to assume that, since the patch and the sheet are thin, these stresses vary linearly with thickness in the patch and the sheet. With these assumptions the distribution of the shear stresses τ_{xz} and τ_{yz} is found to be as in [10]:,

$$\begin{aligned} \tau_{xz} &= f(z)\,\tau_{ax} \\ \tau_{yz} &= f(z)\,\tau_{ay} \end{aligned} \tag{1}$$

where

$$\begin{aligned} f(z) &= 2\,z/t_s \text{ for } 0 \leqslant z \leqslant t_s/z \\ &= 1 \text{ for } t_s/2 \leqslant z \leqslant t_s/2 + t_a \\ &= (t_s/2 + t_a + t_0 - z)/t_0 \text{ for } t_s/2 + t_a \leqslant z \leqslant t_s/2 + t_0 + t_a \end{aligned} \tag{2}$$

and where τ_{ax}, τ_{ay} are the shear stresses in the adhesive and are assumed to be constant through the thickness of the adhesive. The $z = 0$ plane is taken at the mid-surface of the sheet for a doubly reinforced sheet and at the lower surface of the sheet for a singly reinforced sheet. Here

t_o, t_a are the thicknesses of the patch and adhesive respectively while t_s is the sheet thickness in the doubly reinforced case and is twice the sheet thickness in the singly reinforced case. This stress distribution is shown in Figure 1.

Let us consider the case of a doubly reinforced sheet. If the x y axes are at an angle θ to the axes of orthotropy, denoted by x' y', for the composite patch and at an angle $\hat{\theta}$ to the axes of orthotropy, denoted by x'' y'', for the composite sheet then, in the patch

$$\tau_{xz} = G'_{13} (\gamma_{xz} \cos^2 \theta + \gamma_{yz} \sin \theta \cos \theta) - G'_{23} (\gamma_{yz} \sin \theta \cos \theta - \gamma_{xz} \sin^2 \theta) \tag{3}$$

$$\tau_{yz} = G'_{13} (\gamma_{xz} \sin \theta \cos \theta + \gamma_{xz} \sin^2 \theta) + G'_{23} (\gamma_{yz} \cos^2 \theta - \gamma_{xz} \sin \theta \cos \theta) \tag{4}$$

where G'_{13} and G'_{23} are the interlaminar shear moduli of the patch in the x' y' axes system. Here

$$\gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}; \ \gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \tag{5}$$

where u, v, w are displacements in the x, y and z directions respectively.

Substituting for τ_{xz}, and τ_{yz} as given by equation (1) and for γ_{xz} and γ_{yz} as given by equation (5) into equations (3) and (4) yields, after a slight rearrangement of terms,

$$\frac{\partial u}{\partial z} f_1 + \frac{\partial v}{\partial z} f_2 = f(z) \tau_{sy} - f_1 \frac{\partial w}{\partial x} - f_2 \frac{\partial w}{\partial y} \tag{6}$$

$$\frac{\partial u}{\partial z} f_3 + \frac{\partial v}{\partial z} f_1 = f(z)\, \tau_{sx} - f_3 \frac{\partial w}{\partial x} - f_1 \frac{\partial w}{\partial y} \tag{7}$$

where

$$f_1 = (G'_{13} - G'_{23}) \sin \theta \cos \theta$$

$$f_2 = G'_{23} \cos^2 \theta + G'_{13} \sin^2 \theta$$

$$f_3 = G'_{13} \cos^2 \theta + G'_{23} \sin^2 \theta \tag{8}$$

A similar system of partial differential equations may be obtained for the u and v displacements in the sheet viz:

$$h_1 \frac{\partial u}{\partial z} + h_2 \frac{\partial v}{\partial z} = f(z)\, \tau_{sy} - h_1 \frac{\partial w}{\partial x} - h_2 \frac{\partial w}{\partial y} \tag{9}$$

$$h_3 \frac{\partial u}{\partial z} + h_1 \frac{\partial v}{\partial z} = f(z)\, \tau_{sx} - h_3 \frac{\partial w}{\partial x} - h_1 \frac{\partial w}{\partial y} \tag{10}$$

where

$$h_1 = (\hat{G}_{13} - \hat{G}_{23}) \sin \hat{\theta} \cos \hat{\theta}$$

$$h_2 = \hat{G}_{23} \cos^2 \hat{\theta} + \hat{G}_{13} \sin^2 \hat{\theta} \tag{11}$$

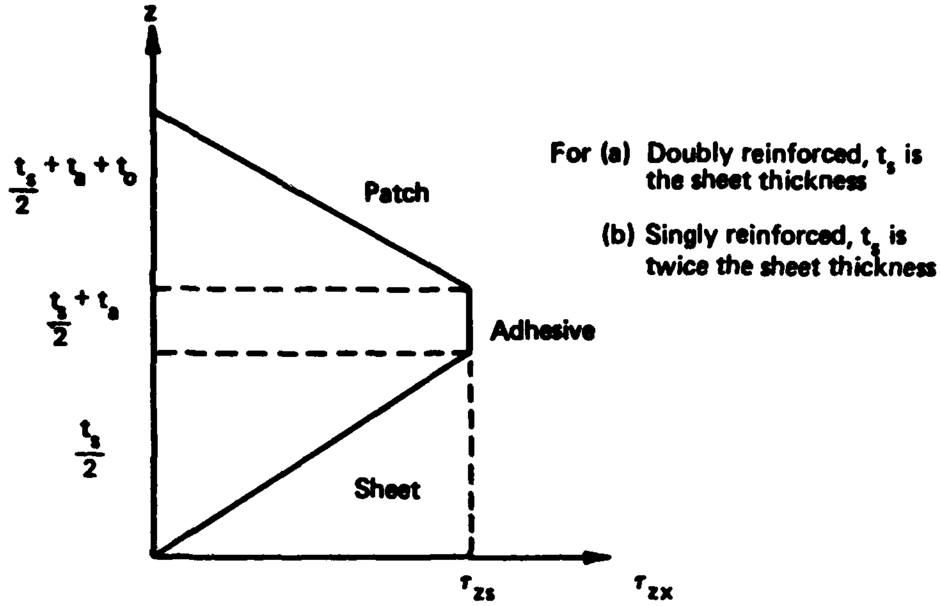
$$h_3 = \hat{G}_{13} \cos^2 \hat{\theta} + \hat{G}_{23} \sin^2 \hat{\theta}$$

and where $\hat{G}_{13}$, $\hat{G}_{23}$ are the interlaminar shear moduli of the sheet in the x'', y'' axes system. Similarly in the adhesive layer we obtain

$$\frac{\partial u}{\partial z} = G_a\, \tau_{sx} - \frac{\partial w}{\partial x} \tag{12}$$

$$\frac{\partial v}{\partial z} = G_a\, \tau_{sy} - \frac{\partial w}{\partial y} \tag{13}$$

If we now assume that the vertical deformation w is independent of z, which is consistent with classical plate theory and Mindlin plate theory, and that the u and v deformations are continuous across the sheet-adhesive interface and the adhesive-patch interface then equations (6), (7), (9), (10), (12) and (13) may be solved to give u and v in terms of w and the shear stresses in the adhesive. Full details of the solution process for the case when $w = 0$ and the sheet is metallic is given in reference [10]. From this solution it is found that the shear stresses τ_{sx} and τ_{sy} may be



DISTRIBUTION OF SHEAR STRESS THROUGH THICKNESS OF PATCHED SHEET

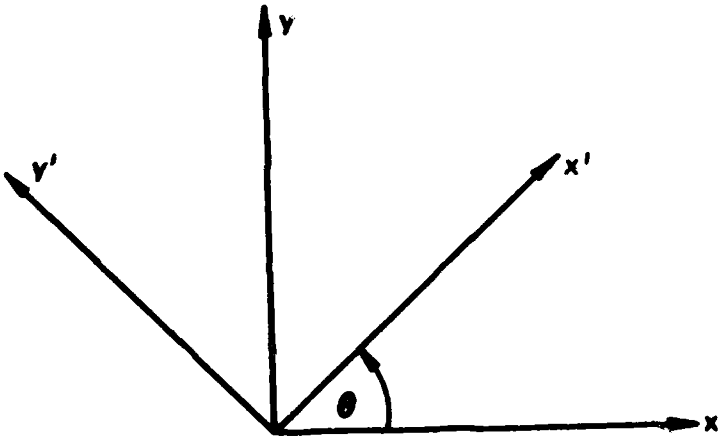


FIG. 1 AXIS SYSTEM IN PATCH

related to the displacements at the midsurface of the patch, which we will denote by u_0, v_0 and w, and the displacements at the midsurface of the sheet, which we will denote by u_s, v_s and w, by the following expression:

$$\tau_{sx} = \left(u_0 - u_s + f_7 \frac{\partial w}{\partial x}\right) f_6/f_8 + \left(v_0 - v_s + f_7 \frac{\partial w}{\partial y}\right) f_4/f_8$$

$$\tau_{sy} = \left(u_0 - u_s + f_7 \frac{\partial w}{\partial x}\right) f_4/f_8 + \left(v_0 - v_s + f_7 \frac{\partial w}{\partial y}\right) f_5/f_8 \tag{15}$$

where for convenience we have denoted

$$f_4 = 3t_0 f_1/8\,(f_2 f_3 - f_1^2) + t_s\, h_1/4\,(h_2 h_3 - h_1^2)$$

$$f_5 = t_a/G_a + 3t_0 f_2/8\,(f_2 f_3 - f_1^2) + t_s\, h_2/4\,(h_2 h_3 - h_1^2)$$

$$f_6 = t_a/G_a + 3t_0 f_3/8\,(f_2 f_3 - f_1^2) + t_s\, h_3/4\,(h_2 h_3 - h_1^2)$$

$$f_7 = (t_a + t_s/2 + t_0/2)$$

$$f_8 = f_5 f_6 - f_4^2 \tag{16}$$

For a singly reinforced sheet, i.e. a patch on one side only, the term $t_s/4$ in the expressions for f_4, f_5 and f_6 is replaced by $3t_s/8$.

In the specific case when bending effects are negligible and when the sheet is isotropic the analysis reduces to that given by the authors in [10]. Furthermore in the case when $G'_{13} = G'_{23}$ ($= G_0$) and $G_{23} = G_{13}$ ($= G_s$), which occurs when the sheet and the patch are transversely isotropic we have

$$f_4 = f_1 = h_1 = 0, f_2 = f_3 = G_0,\ h_2 = h_3 = G_s,\ f_5 = f_6 = t_a/G_a + t_s/_4G_s + 3t_0/_8G_0 \tag{17}$$

so that

$$\tau_{sx} = \left(u_0 - u_s + \left(t_a + \frac{t_s}{2} + \frac{t_0}{2}\right)\frac{\partial w}{\partial x}\right) \bigg/ \left(\frac{t_a}{G_a} + \frac{3t_0}{8G_0} + \frac{t_s}{4G_s}\right) \tag{18}$$

$$\tau_{sy} = \left(v_0 - v_s + \left(t_a + \frac{t_s}{2} + \frac{t_0}{2}\right)\frac{\partial w}{\partial y}\right) \bigg/ \left(t_a/G_a + 3t_0/8G_0 + t_s/4G_s\right) \tag{19}$$

Indeed in the special case when $\partial w/\partial x = \partial w/\partial y = 0$ equations (18) and (19) coincide with the expression given, without detailed proof, in [12]. Furthermore, inspecting equations (14) and (15) we see that in order for τ_{sx}, τ_{sy} to be zero we require

$$v_0 - v_s = -\left(t_a + \frac{t_s}{2} + \frac{t_0}{2}\right)\frac{\partial w}{\partial y} \tag{20}$$

$$u_0 - u_s = -\left(t_a + \frac{t_s}{2} + \frac{t_0}{2}\right)\frac{\partial w}{\partial x} \tag{21}$$

which is in agreement with classical plate theory.

3. ELEMENT STIFFNESS MATRIX

Having thus obtained the relationship between the adhesive shear stress and the displacements in the patch and the sheet it is a relatively simple task to formulate the stiffness matrix for an "adhesive" element. We begin by adopting the standard assumption governing the sheet and patch [10, 12] viz:

The finite element model for the sheet and the patch assumes a state of plane stress.

With this assumption it is first necessary to determine the strain energy due to the shear deformation in an element of the structure viz:

$$V = \tfrac{1}{2} \iiint \left(\tau_{xz}\,\gamma_{xz} + \tau_{yz}\,\gamma_{yz}\right) dz dx dy \tag{22}$$

Here the z intergration is over the total thickness of the sheet, the adhesive and the patch while the x and y integration is over the area of the element. Although in the following analysis we will only consider an element which is triangular in planform (see Fig. 2) the formulation may easily be extended so as to allow the element to be of any standard shape.

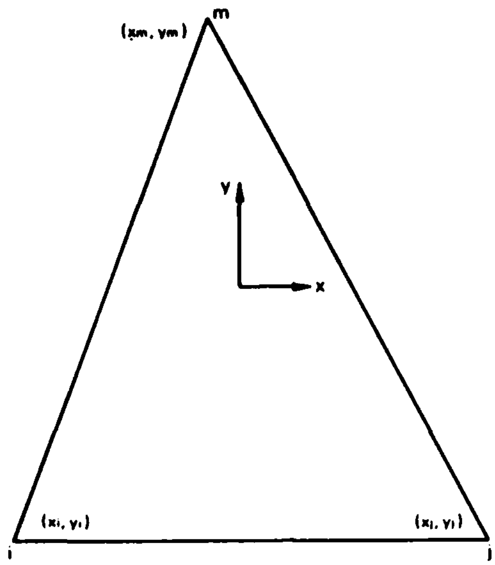


FIG. 2 ELEMENT OF ADHESIVE

Using the same notation as in the book by Zenkiewicz [13] we define a vector **f** such that

$$\mathbf{f}^T = (u_0, v_0, u_s, v_s, w, \theta_x, \theta_y)$$
$$= (N\delta)^T \tag{23}$$

where as explained in [13] page 18, the components of N are generalised functions of position and

$$\delta^T = (\delta_i^T, \delta_j^T, \delta_m^T) \tag{24}$$

where

$$\delta_i^T = (u_{0i}, v_{0i}, u_{si}, v_{si}, w_i, \theta_{xi}, \theta_{yi}) \tag{25}$$

Here, as in [13], $\theta_x = - \partial w/\partial y$, $\theta_y = \partial w/\partial x$.

The strain vector γ may after some manipulation be expressed as

$$\gamma^T = (\gamma_{xz}, \gamma_{yz}) = (D\tau)^T \tag{26}$$

where

$$\tau^T = (\tau_{zx}, \tau_{zy}) \tag{27}$$

and the matrix D, which depends on the thickness coordinate z, is given by:

$$D = \frac{f(z)}{(h_1^2 - h_2 h_3)} \begin{pmatrix} -h_2 & h_1 \\ h_1 & -h_3 \end{pmatrix}$$

in the patch

$$= \frac{1}{G_a}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \text{ in the adhesive} \tag{28}$$

$$= \frac{f(z)}{(f_1^2 - f_2 f_3)}\begin{pmatrix}-f_2 & f_1\\ f_1 & -f_3\end{pmatrix} \text{ in the sheet}$$

and where $f(z)$ is as defined in equation (1).

Making use of equations (14) and (15) and (23) we find that τ may be written as

$$\tau = AN\delta \tag{29}$$

where

$$A = 1/f_8\begin{bmatrix} f_6, f_4, -f_6\ -f_4, 0, -f_4 f_7, f_6 f_7\\ f_4, f_5, -f_4, -f_5, 0, -f_5 f_7, f_4 f_7\end{bmatrix} \tag{30}$$

Substituting for τ, as given in (29) into equation (26) gives

$$\gamma = D\,A\,N\delta \tag{31}$$

If we now substitute equations (1), (29), and (31) into the strain energy expression we obtain

$$V = \tfrac{1}{2}\iint (AN\delta)^T\left(\int f(z)\, D dz\right) AN\delta\, dxdy \tag{32}$$

The stiffness matrix K^e may now be obtained, as outlined in (13), by differentiating V with respect to δ and is given by

$$K^e = \iint (AN)^T\left(\int f(z)\, D dz\right) AN\, dxdy \tag{33}$$

As before the double integration is over the area of the element while the integration with respect to z is over the thickness of the structure.

This formulation looks quite complex but in the case when the interlaminar shear moduli of the patch (and the sheet) are equal it simplifies considerably. For example, let the structure be patched on both sides and let $G'_{13} = G'_{23}\,(= G_0)$ and $\hat{G}_{13} = \hat{G}_{23}\,(= G_s)$. This gives

$$K^e = 2\left((t_a/G_a + t_0/3G_0 + t_s/6G_s)\right)\iint (AN)^T\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} AN\, dxdy \tag{34}$$

where A has simplified to

$$A = \begin{pmatrix}1, 0, -1, 0, 0, 0, f_7\\ 0, 1, 0, -1, 0, -f_7, 0\end{pmatrix}\Big/\left(\frac{t_a}{G_a} + \frac{t_s}{4G_s} + \frac{3t_0}{8G_0}\right)$$

and as before $f_7 = t_a + t_s/2 + t_0/2$. It is important to note that as the adhesive thickness tends to zero (i.e. $t_a \to 0$) the stiffness matrix K^e tends to a constant and non zero value. This should enable the present approach to be used to model internal delaminations as well as the repair problems which will be considered here.

This approach coincides with the analysis presented in [10] when $\partial w/\partial x = \partial w/\partial y = 0$ and the sheet is isotropic. Furthermore when $\partial w/\partial x = \partial w/\partial y = 0$ but the sheet is a composite laminate the functional form of the shear stresses τ_{sx} and τ_{sy}, as given by equations (14) and (15), is similar to that given in [10]. The difference is that the expressions for f_4, f_5, and f_6 given in the present paper are more complex than those given in [10] for an isotropic sheet.

4. ILLUSTRATIVE EXAMPLES

Let us now consider the repair of damaged fibre composite panels. The basic ply is a graphite/epoxy (Narmco Thornel T300/5208 with the following material properties $E_1 = 141\cdot0 \times 10^3$ MPa, $\nu_{12} = 0\cdot31$, $G_{12} = 5\cdot18 \times 10^3 MPa$, $E_1/E_2 = 14\cdot96$. Since it is often stated that composites are relatively notch insensitive two particular kinds of damage were considered. The first is a centrally located crack $38\cdot1$ mm long in a graphite epoxy panel of dimensions 508 mm $\times$

635 mm $\times$ 2·29 mm. The second kind of damage is a centrally located hole with a diameter of 38·1 mm in the same panel. Various panel lay ups were considered but to enable a simple comparison of the numerical results the panel thickness was taken as 2·29 mm for each lay up. The moduli of the panel were taken to be as for a symmetric lay up of either

(1) $(0/\pm 45/90)_s$
(2) $(0/90)_s$
(3) $(0/\pm 45)_s$
(4) $(\pm 45)_s$
(5) $(0)_s$

These laminates were chosen because of their similarity to skin lay ups in existing or planned military aircraft. In each case the panels were assumed to be subjected to a uniform tensile stress at these edges. Because of the symmetrical nature of these problems only one half of the structure was analysed.

The finite element mesh for the cracked panel consisted of 147 constant strain triangles, 142 quadrilateral elements and a special crack element, (see reference [10].)

The mesh for the panel containing a hole consisted of approximately the same number of elements.

To these models was added a finite element model of the various bonded repairs (i.e. patches). A patch was assumed to be placed on both sides of the panel. Each patch is stepped in thickness and has dimensions 152 mm $\times$ 50·8 mm with a maximum thickness of 0·762 mm, which occurs in the middle of the patch; see Figure 3. The patches considered were:

(*a*) Titanium
(*b*) A unidirectional graphite/epoxy patch with the fibres in the direction of the load.
(*c*) A graphite/epoxy laminate with the same moduli as the sheet
(*d*) A quasi-isotropic (e.g. $0/\pm 45/90$) boron epoxy laminate.

The finite element model of these patches consisted of 150 constant strain triangles and 150 of the adhesive elements described above. In this problem the analysis simplified considerably since the sheet was patched on both sides and the load was acting in the plane of the midsurface of the sheet. As a result the structure did not experience bending effects, i.e.

$$w = \frac{\partial w}{\partial x} = \frac{\partial w}{\partial y} = 0 \tag{36}$$

This in turn simplified the formulation of the adhesive stiffness matrix, since it was now only necessary to specify the way in which the u and v displacements varied in the adhesive element. In this paper we follow the approach given in [10] using a triangular element and taking

$$u = ((a_i + b_i x + C_i y)\, u_i + (a_j + b_j x + c_j y)\, u_j + (a_m + b_m x + C_m y)\, u_m)/2\Delta \tag{37}$$

and a similar expression for v where as in [13]

$$a_i = x_j y_m - x_m y_j,\ b_i = y_m - y_j,\ C_i = x_m - x_j \tag{38}$$

and a_j, a_m, b_j, b_m, c_j and c_m are obtained by a cyclic permutation of i, j, m. Here (x_i, y_i), (x_j, y_j) and (x_m, y_m) are the coordinates of the corners of the element, Δ is the area and u_i, v_i are the displacements at the ith node (see Fig. 2).

The effect that patches have on the damaged panels is described below where we first consider the repair to the cracked panel using either patch (*a*) or patch (*b*). In Table 1 we see the ratio of the stress intensity factors K_{ip}/K_{iu}, where K_{ip} and K_{iu} are the values of the stress intensity factor after and before patching respectively. The values obtained for K_{iu} coincided with those previously published and which differ from the value obtained for an isotropic sheet by only a few percent.

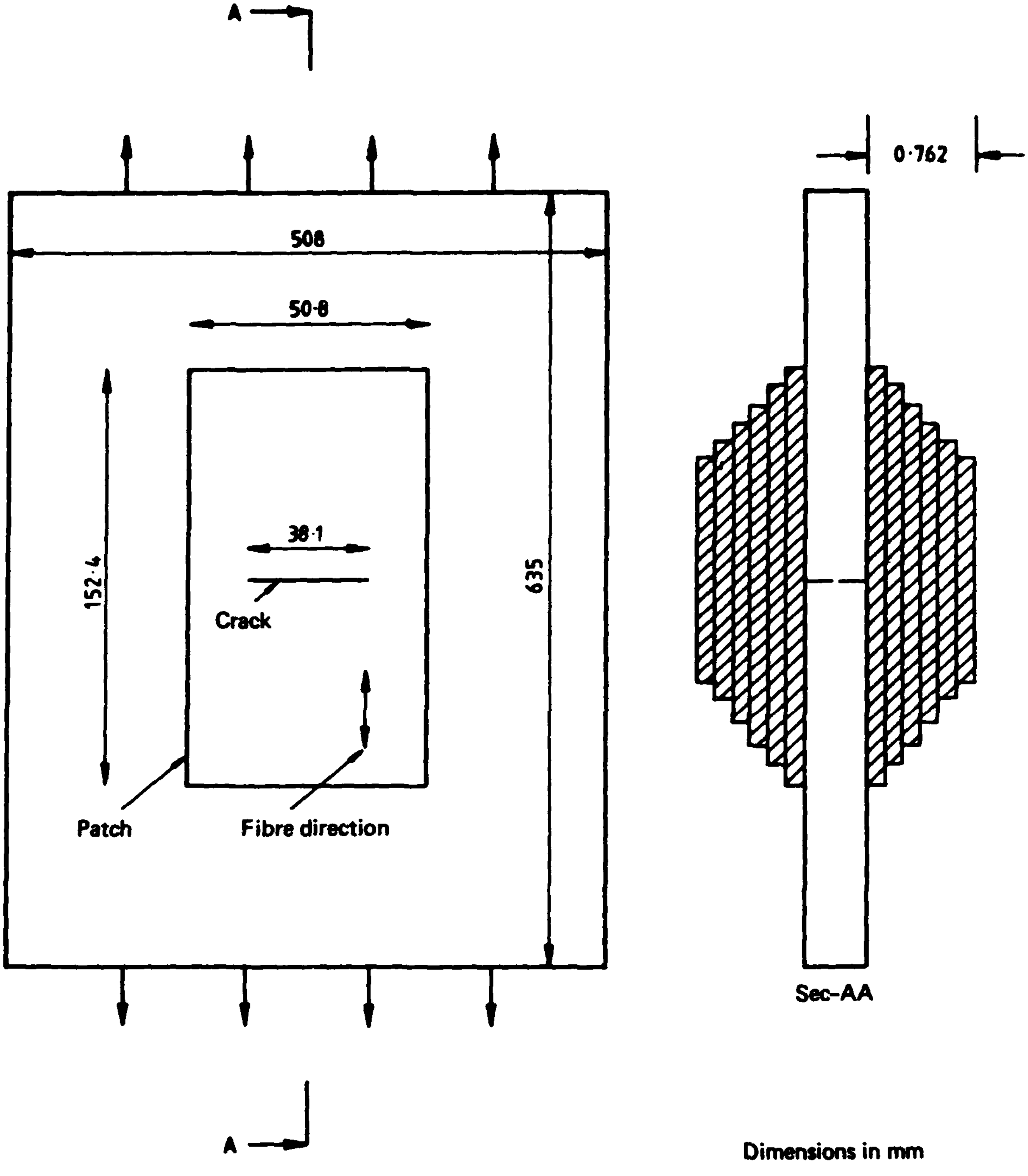


FIG. 3 PATCHED PANEL

Table 1. Ratios of stress intensity factors

K_{Ip}/K_{Iu}

Sheet Material	Repair Material	
	Unidir. graphite/epoxy	Titanium
$(0/\pm 45/90)_s$	0·16	0·14
$(0/90)_s$	0·14	0·14
$(0/\pm 45)_s$	0·17	0·16
$(\pm 45)_s$	0·11	0·11
$(0)_s$	0·18	0·17

From this table we see that as a repair to cracked graphite epoxy sheets the unidirectional graphite laminate, with the fibres perpendicular to the crack, viz in the direction of the load, and the titanium are almost equally effective in reducing the stress intensity factor at the crack tip. This is consistent with the results given in [11] which deals with the repair of a crack of the same length in an aluminium panel of the same dimensions as considered here. Indeed in [11] the authors show that a slight increase in the stiffness of the patch does not greatly reduce the stress intensity factor. Nevertheless at first glance it is surprising that the titanium, which has a Young's modulus of $106 \cdot 7 \times 10^3$ MPa, is as effective as the unidirectional graphite epoxy laminate with $E_{11} = 141 \cdot 0 \times 10^3$ MPa. The explanation of this phenomenon is partly due to the fact that, as indicated above, the rate of reduction of the stress intensity factor with increasing stiffness is small and partly due to the fact that the transverse shear modulus for the titanium ($G_{13} = G_{23} = 40 \cdot 5 \times 10^3$ MPa) is much greater than that of the graphite/epoxy ($G_{23} = G_{13} = 5 \cdot 2 \times 10^3$ MPa). In titanium the crack opening displacement is resisted by virtually the entire thickness of the titanium, whereas in graphite the low value of its transverse shear modulus results in the plies being unequally stressed with the plies closest to the surface providing the most resistance to the opening of the crack.

Although here we have confined our attention to unidirectional graphite patches there are many reasons for using a uni-directional boron patch to repair cracks. One of the main reasons is that standard eddy current inspection procedures may be used to inspect for crack growth under a boron patch. For further details concerning the advantages of boron/epoxy over carbon/epoxy laminates as repair material see [7].

Although we have seen that both repairs give rise to approximately the same reduction in the stress intensity factors there are several reasons why a unidirectional repair should be used in preference to a titanium patch. Indeed the reasons for recommending use of a uni-directional laminate to repair cracked composite sheets are the same as for recommending the use of a unidirectional laminate to repair cracked metallic sheets and are given in [7, 8]. These are best illustrated by considering the recent repair to cracks in the lower wing skin of Mirage III aircraft in service with the Royal Australian Air Force; see reference [8]. This repair was a unidirectional boron/epoxy laminate with the fibres perpendicular to the crack and was designed by the authors. In this case the crack lay in vicinity of the spar and the root rib intersection. The use of a titanium patch, with its parasitic stiffness in the direction parallel to the crack, could easily have changed the strain distribution in the spar, which was itself stress critical. However by using the unidirectional boron laminate the strain in the spar, with a cracked but patched wing skin, was restored to the value of the strain in a spar with an uncracked wing skin; see reference [8].

Let us now turn our attention to the repair of holes in graphite/epoxy laminates. In Tables 2 and 3 are given the ratio of the stress concentration factors K_{TP}/K_{TU} and the ratio of the maximum fibre stress to the applied stress σ_p/σ. Here K_{TP} and K_{TU} are the stress concentration factors in the sheet after and before patching respectively. The values obtained for K_{TU} coincided with those previously given in [3].

Table 2. Ratio of Stress Concentration Factors K_{TP}/K_{TU}

Panel laminate Patch	0/±45	±45	0/90/±45	0/90	0
a	0·32	0·32	0·33	0·32	0·26
b	0·30	0·28	0·32	0·26	0·26
c	0·43	0·54	0·46	0·40	0·26
d	0·36	0·32	0·32	0·37	0·31

Table 3. Ratio of Patch Stress to Sheet stress σ_p/σ

Panel laminate Patch	0/±45	±45	0/90/±45	0/90	0
a	3·05	4·2	2·69	2·25	2·32
b	3·94	5·4	3·46	2·86	3·08
c	2·43	1·73	1·91	2·02	3·08
d	2·73	3·82	3·46	2·01	2·12

From Tables 2 and 3 we see that when repairing holes in composite sheets under uniaxial loading it is best to use a unidirectional laminate with the fibres in the direction of the load. Indeed one may generalize this and say that if, under an arbitrary system of loads, one knows the direction of the principal stress which is primarily responsible for failure then it is best to use as a repair a unidirectional laminate with the fibres in the direction of the principal stress. In general however one may not know this direction and in this case the use of a unidirectional laminate as a repair could be unconservative. Indeed in this situation the use of a bonded titanium, or a quasi-isotropic boron patch as a repair is quite common [5]. Furthermore consulting Table 2 we see that the titanium and the quasi-isotropic boron patches give approximately the same ratio of the stress concentration factors which is significantly lower than that obtained using a patch with the same lay up as the skin material.

As a result since bonded repairs often take place either in the field [5, 8] where the direction of the principal stress, may not be available, or in a repair depot the recommended repairs to holes in composite sheets is either a bonded titanium or a quasi-isotropic boron patch. On the other hand if the panel contains a distinct crack the recommended repair is a unidirectional laminate with the fibres perpendicular to the crack.

5. CONCLUSION

An advanced finite element method for analysing the repair of damaged composite laminates has been developed. Subsequent analysis has shown that either a titanium or a quasi-isotropic boron repair is equally suited to the repair of holes while a unidirectional laminate, with its fibres perpendicular to the crack, is best suited to the repair of cracks in composite laminates.

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